Sociophysics

Probability of Inconsistency in Opinion Dynamics

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Introduction

Sociophysics is an interdisciplinary field which applies the methodology of statistical physics to questions in sociology. Opinion dynamics provides agent-based models to study processes of belief revision in societies. The agents update their current opinions by averaging the belief states of other agents in the community. When the agents hold opinions on logically related issues, the updating process may give rise to inconsistent belief states. Our goal is to calculate the probability for this to occur.

We model the agents' beliefs as 'theories about the world' and assume that each agent holds a consistent theory initially. They update their opinion by taking into account the opinions of others that are not too different (quantified by a bound of confidence). We calculate the probability for an agent to arrive at an inconsistent theory after a single update, using 3 parameters:

- M = number of propositions
- N = number of agents
- D = bound of confidence

Example of Theories for M=2

M=2 Two independent propositions

E.g., $m = 0$: "It will rain tomorrow"

$m = 1$: "There exist magnetic monopoles"

	$m=1$	$m=0$
$w=0$	0	0
$w=1$	0	1
$w=2$	1	0
$w=3$	1	1

$w_{max} = 2^M = 4$ possible worlds

Combinations of propositions being true (1) or false (0).

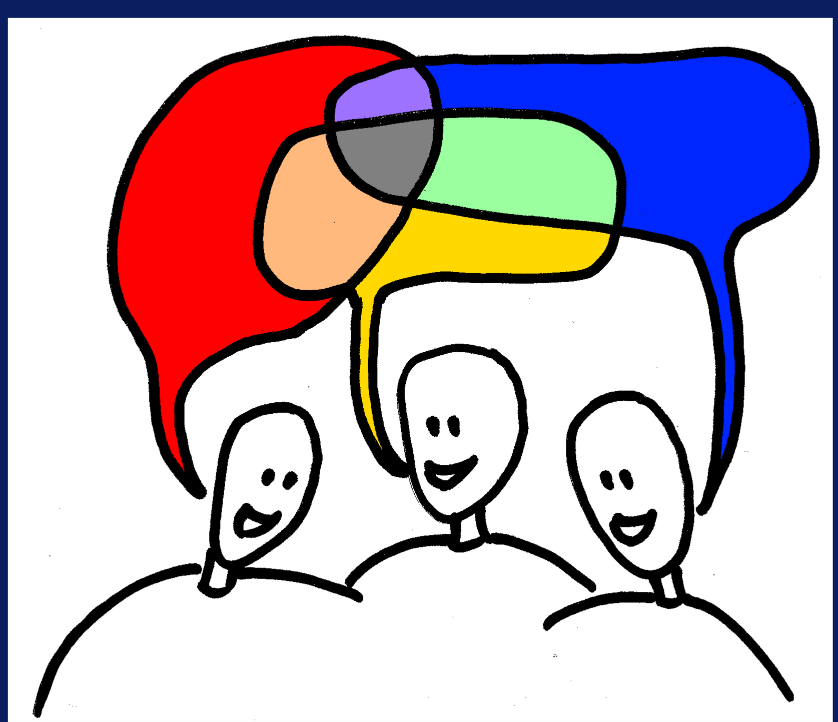
$t_{max} = 2^{2^M} = 16$ theories

Combinations of possible worlds being allowed (1) or rejected (0). There is one inconsistent theory, $t = 0$. It rejects *all* possible worlds, but the actual world *must* be among them.

n_t = # agents holding theory t

	$w=3$	$w=2$	$w=1$	$w=0$	opinion profile
$t=0$	0	0	0	0	n_0
$t=1$	0	0	0	1	n_1
$t=2$	0	0	1	0	n_2
$t=3$	0	0	1	1	n_3
$t=4$	0	1	0	0	n_4
$t=5$	0	1	0	1	n_5
$t=6$	0	1	1	0	n_6
$t=7$	0	1	1	1	n_7
$t=8$	1	0	0	0	n_8
$t=9$	1	0	0	1	n_9
$t=10$	1	0	1	0	n_{10}
$t=11$	1	0	1	1	n_{11}
$t=12$	1	1	0	0	n_{12}
$t=13$	1	1	0	1	n_{13}
$t=14$	1	1	1	0	n_{14}
$t=15$	1	1	1	1	n_{15}

Example of Inconsistency



Suppose: **M=2, N=3, D=2**

Initial theory of Agent A: $t = 1$ (0 0 0 1)

Initial theory of Agent B: $t = 8$ (1 0 0 0)

Initial theory of Agent C: $t = 2$ (0 0 1 0)

All theories differ from each other by two bits; this is within the bound of confidence (D=2). The average opinion is (⅓ 0 ⅓ ⅓), which gets rounded to (0 0 0 0), or $t = 0$.

Analytical Expression

Probability for an agent to update to $t = 0$

$$F_{AG}(M,N,D) = \sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\cdots\sum_{n_{t_{max}-2}=0}^{N-(n_1+n_2+\cdots+n_{t_{max}-3})}\frac{N!}{n_0!n_1!\cdots n_{t_{max}-1}!}\frac{1}{(t_{max}-1)^N}\sum_{t=0}^{t_{max}-1}\left(\frac{n_t}{N}\prod_{w=0}^{w_{max}-1}\mathrm{INV}[\langle B_w(t)\rangle]\right)$$

$B_w(t)$: w^{th} bit of theory t

$\langle B_w(t)\rangle$: bounded average of w^{th} bit

$\mathrm{INV}[\langle B_w(t)\rangle]$: 0 if average bit is >½, 1 if average bit is < ½

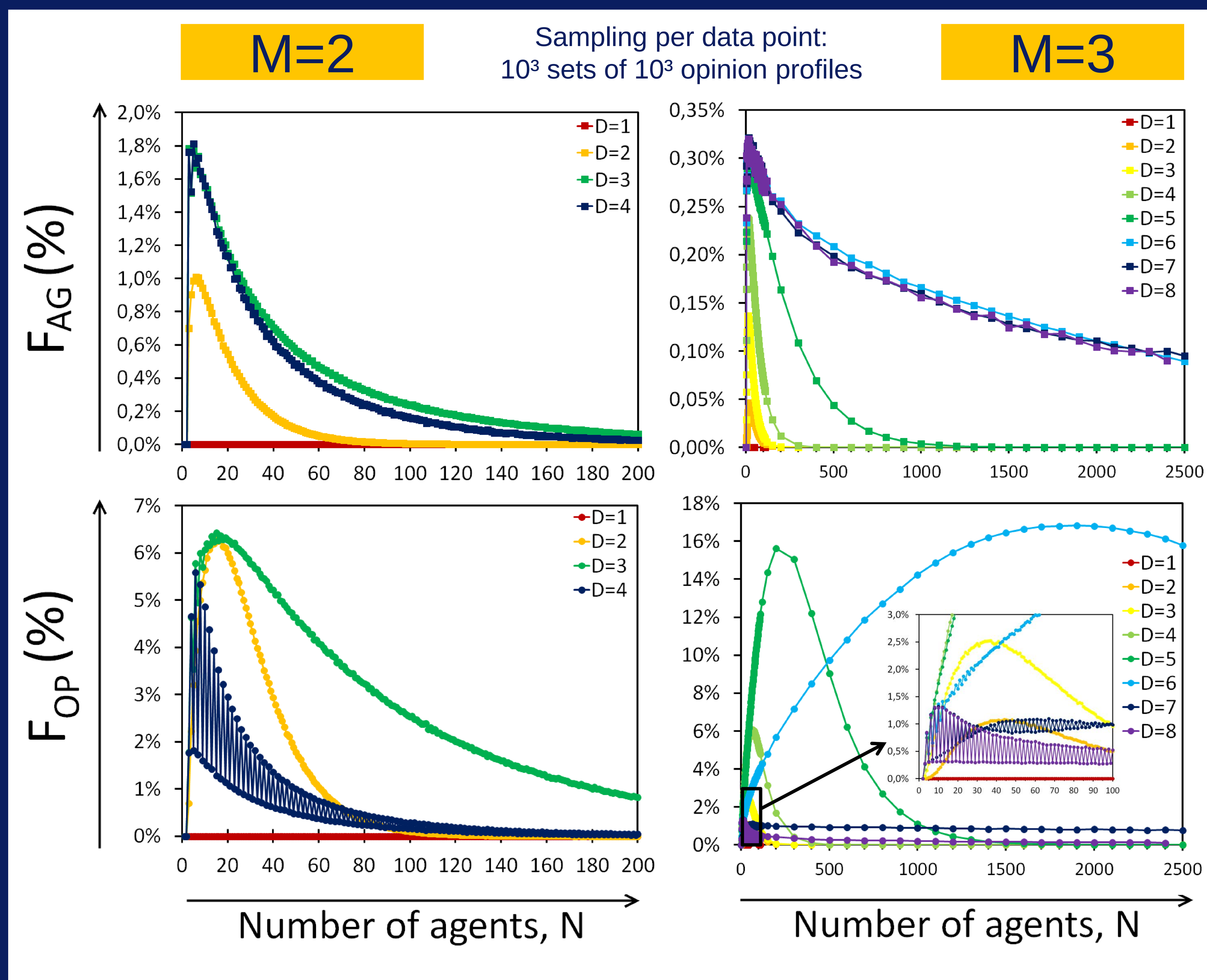
= 0 ⇒ ZUP = 0

> 0 ⇒ ZUP = 1

$$F_{OP}(M,N,D) = \sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}\cdots\sum_{n_{t_{max}-2}=0}^{N-(n_1+n_2+\cdots+n_{t_{max}-3})}\frac{N!}{n_0!n_1!\cdots n_{t_{max}-1}!}\frac{1}{(t_{max}-1)^N}\mathrm{ZUP}(M,N,D,\vec{n})$$

Probability that at least one agent in group updates to $t = 0$

Computational Results



Conclusions

Probability to update to inconsistent theory **> 0%** (apart from trivial cases: N=2 or M=1), but **< 2%**

Intermediate D: highest max(F_{OP})

Even vs odd group size: "wobble" in graphs

Reference: S. Wenmackers, D.E.P. Vanpoucke, I. Douven "Determining the Probability of Inconsistencies in Theory Updating under Bounded Confidence", submitted to *Journal of Mathematical Sociology* (2010).

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